

QTS0103 Business Mathematics — CA1 Complete Guide

Continuous Assessment 1 • Individual Assignment (占总成绩的30%) • 解题指导文件

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§ 0 使用说明 How to Use This Guide

1. 本文件仅供参考。先独立完成，再用本指南核对步骤顺序和格式。直接抄写不仅有 plagiarism (剽窃) 风险，而且 CA2 和期末考同类型题你会做不出来。
2. 步骤即分数。卷面明确写着: "Marks will be awarded for workings shown. No marks will be awarded if there are no workings." (有过程才有分，没过程零分)。本指南中每一行推导都对应 lecture slides 的标准写法，不要跳步。
3. 必须用讲义方法。每题末尾都注明: "Alternate workings will not warrant full marks" (用其他方法不给满分)。例如 Q2(c) 指定 factorisation，就不能用 quadratic formula。
4. 最终提交前用 § 6 的 checklist 自查一遍。

§ 1 模块与提交要求 Module & Submission Requirements

ITEM 项目	REQUIREMENT 要求
Weighting 占比	30% of module grade (个人作业 Individual Assignment)
Format 格式	Typewritten in Microsoft Word, 字体 Times New Roman, size 12
Submission 提交	Online via Canvas, 截止日期见 Canvas; late submission 有罚则
Identity 身份信息	要写姓名 + student number
Scope 范围	共 3 题，每题 10 分，全部必答 (Total 30 marks)
Workings 过程	必须展示完整过程；无过程 = 0 分；必须用 lecture slides 的标准方法
Integrity 诚信	Plagiarism is a serious offence, 一律上报 administration
Retention 留底	自留一份副本，答卷不退回

Word 打公式的建议：用 Insert → Equation (快捷键 Alt +=) 逐行输入，每一步等式单独一行、等号对齐。不要截图贴数学式。

Q1(A)(I) • LINEAR EQUATION 线性方程

[2 MARKS]

Solve $8x - 5 = 22 - 4x$

Key point: 移项归类 (collect like terms) ——未知项归左, 常数归右, 每步单独写一行。

$$8x - 5 = 22 - 4x$$

$$8x + 4x = 22 + 5 \quad (\text{add } 4x \text{ and } 5 \text{ to both sides})$$

$$12x = 27$$

$$x = 27/12 = \frac{9}{4} \quad (= 2.25)$$

Q1(A)(II) • QUADRATIC INEQUALITY 二次不等式

[2 MARKS]

Solve $x^2 - 4x - 21 \leq 0$ Method: factorise → find roots → sign analysis. 先因式分解, 找到两根, 再判断抛物线开口方向决定解区间。系数为正 → 开口向上 → " ≤ 0 " 的解在两根之间 (between the roots)。

$$x^2 - 4x - 21 \leq 0$$

$$(x + 3)(x - 7) \leq 0 \quad (\text{factorise: } -21 = (+3)(-7), +3-7 = -4 \checkmark)$$

$$\text{Critical values: } x = -3, x = 7$$

Since the coefficient of x^2 is positive (parabola opens upwards), $(x + 3)(x - 7) \leq 0$ when x lies between the roots.

$$\therefore -3 \leq x \leq 7$$

建议附一句 sign test 或简易数轴/抛物线草图说明, 呼应 lecture 的呈现方式。答案必须写成 $-3 \leq x \leq 7$ 的区间形式, 不是两个孤立的根。

Q1(B)(I) • COST / REVENUE / PROFIT FUNCTIONS

[3 MARKS]

Fixed cost \$120,000/month • Variable cost \$15/unit • Selling price \$35/unit套 Lecture 4 的三个标准函数。 $C(x) = cx + F$ (variable cost $\times$ 数量 + fixed cost), $R(x) = sx$, $P(x) = R(x) - C(x)$ 。开头必须交代变量定义: Let x be the number of toy cars produced and sold per month。Let x = number of toy cars produced and sold per month.

$$\text{Cost function: } C(x) = 15x + 120,000$$

$$\text{Revenue function: } R(x) = 35x$$

$$\begin{aligned} \text{Profit function: } P(x) &= R(x) - C(x) \\ &= 35x - (15x + 120,000) \\ &= 20x - 120,000 \end{aligned}$$

更多支持, 访问: simhelper.site

Find the break-even output and corresponding revenue

At the break-even point, $R(x) = C(x)$ (lecture 标准开场句, 照写)。解出 break-even quantity 后, 必须代回 $R(x)$ 求 break-even revenue——这一步是题目明确要的 "corresponding revenue"。

At the break-even point, $R(x) = C(x)$:

$$35x = 15x + 120,000$$

$$20x = 120,000$$

$$x = 6,000$$

Substituting $x = 6,000$ into the revenue function:

$$R(6,000) = 35(6,000) = 210,000$$

∴ Break-even quantity = 6,000 toy cars per month

Break-even revenue =

替代路线 (同为讲义方法): 令 $P(x) = 0$, 即 $20x - 120,000 = 0 \Rightarrow x = 6,000$ 。两条路都可, 选一条写清楚即可。

Q2 Depreciation, Equilibrium & Intersection

10 marks

Purchase \$50,000 • Residual value \$2,000 after 10 years • Find $V(t)$

Annual depreciation = (initial value - residual value) ÷ useful life. 直线折旧就是一条线性函数: 截距 = 购入价, 斜率 = -每年折旧额。

$$\begin{aligned} \text{Annual rate of depreciation} &= (50,000 - 2,000) / 10 \\ &= 48,000 / 10 \\ &= \$4,800 \text{ per year} \end{aligned}$$

$$\therefore V(t) = 50,000 - 4,800t, \quad \blacksquare \leq t \leq \blacksquare$$

注意写上定义域 $\blacksquare \leq t \leq 10$ ——折旧模型只在使用年限内有效, lecture 里都有标注。

State the value of the car at the end of the fifth year

$$\begin{aligned} V(5) &= 50,000 - 4,800(5) \\ &= 50,000 - 24,000 \\ &= \blacksquare \end{aligned}$$

Demand: $p = -0.3x^2 + 20x + 40$ • **Supply:** $p = 0.2x^2 + 3x + 10$

At market equilibrium, demand = supply. 联立两式 → 化为标准二次方程 → 用 quadratic formula 求根 → 拒绝负根并写明理由 ("as x cannot be negative", 这是 lecture Example 4.10 的固定句式) → 代回任一函数求 equilibrium price.

At market equilibrium, equate the demand and supply functions:

$$-0.3x^2 + 20x + 40 = 0.2x^2 + 3x + 10$$

$$0.5x^2 - 17x - 30 = 0$$

By the quadratic formula ($a = 0.5$, $b = -17$, $c = -30$):

$$x = [17 \pm \sqrt{(-17)^2 - 4(0.5)(-30)}] / (2 \times 0.5)$$

$$= [17 \pm \sqrt{289 + 60}] / 1$$

$$= 17 \pm \sqrt{349}$$

$$x = \blacksquare \text{ or } x = -\blacksquare \text{ (rejected, as } x \text{ cannot be negative)}$$

Substituting $x = 35.68$ into the supply function:

$$p = 0.2(35.68)^2 + 3(35.68) + 10$$

$$\approx \blacksquare$$

$$\therefore \text{Equilibrium quantity} \approx \blacksquare$$

$$\text{Equilibrium price} \approx \blacksquare$$

两个细节: ① x 的单位是 units of thousands (千件), 最终陈述时要换算或注明; ② 根不是整数, 中间过程保留 4 位小数、最终答案保留 2 位小数 (2 d.p.), 避免累积舍入误差。

$f(x) = 3x^2 - 5x - 2$ and $g(x) = x^2 + x - 6$

The question specifies "by using factorisation" —— 必须因式分解, 用 quadratic formula 会被扣分。求交点 = 令 $f(x) = g(x)$ 解 x , 再代回任一函数求 y , 最后以坐标点 (x, y) 作答。

At the point(s) of intersection, $f(x) = g(x)$:

$$3x^2 - 5x - 2 = x^2 + x - 6$$

$$2x^2 - 6x + 4 = 0$$

$$x^2 - 3x + 2 = 0 \quad (\text{divide both sides by } 2)$$

$$(x - 1)(x - 2) = 0 \quad (\text{factorisation})$$

$$x = \blacksquare \text{ or } x = \blacksquare$$

$$\text{When } x = \blacksquare: g(1) = (1)^2 + 1 - 6 = \blacksquare$$

$$\text{When } x = \blacksquare: g(2) = (2)^2 + 2 - 6 = \blacksquare$$

$$\therefore \text{The points of intersection are } (\blacksquare, \blacksquare) \text{ and } (\blacksquare, \blacksquare).$$

自查: 代入 $f(x)$ 验证 —— $f(1) = 3 - 5 - 2 = -4 \checkmark$, $f(2) = 12 - 10 - 2 = 0 \checkmark$ 。两个交点都要写, 漏一个丢分。

Given $\log x = p$ (底数为 10 的 common logarithm)

每一小问都要明确引用所用的 **log law** (power law / product law)，这是 "full set of rule(s) shown in lecture slides" 的要求。

$$\begin{aligned} \text{(i)} \quad \log x^2 &= 2 \log x && \text{(power law)} \\ &= \blacksquare \end{aligned}$$

$$\begin{aligned} \text{(ii)} \quad \log(10\sqrt{x}) &= \log 10 + \log x^{(1/2)} && \text{(product law)} \\ &= 1 + (1/2) \log x && \text{(power law; } \log 10 = 1) \\ &= \blacksquare \end{aligned}$$

$$\begin{aligned} \text{(iii)} \quad \text{Since } \log x &= p \text{ (base 10),} \\ x &= 10^p && \text{(definition of logarithm)} \\ \therefore x^3 &= \blacksquare \end{aligned}$$

(iii) 考的是 logarithmic $\leftrightarrow$ exponential form conversion (对数式与指数式互化)： $\log x = p \Leftrightarrow x = 10^p$ 。答案必须是 p 的表达式，即 $\blacksquare$ 。

$Q(t) = Q_0 e^{(kt)}$ • $Q(0) = \$100,000$ • $Q(10) = \$200,000$ • Show $k = (\ln a)/b$

两步："initially" $\Rightarrow t=0$ 定出 Q_0 ；再用 $t=10$ 的条件解 k 。解 e 的方程用 natural logarithm ($\ln$)，且要写出 "taking natural logarithm on both sides" 这一步。这是 "show that" 题——每一步都不能省，最后必须落在 $k = \ln 2 / 10$ 并点明 a 、 b 。

$$\text{When } t = 0 \text{ (initially): } Q(0) = Q_0 e^0 = Q_0 = 100,000$$

When $t = 10$:

$$\begin{aligned} 100,000 e^{(10k)} &= 200,000 \\ e^{(10k)} &= 2 \end{aligned}$$

Taking natural logarithm on both sides:

$$\begin{aligned} \ln e^{(10k)} &= \ln 2 \\ 10k &= \ln 2 \\ k &= \blacksquare \end{aligned}$$

$$\therefore k = \blacksquare \text{ with } a = 2, b = 10. \text{ (shown)}$$

Find $Q(20)$

代 $t = 20$ 。亮点步骤： $e^{(2 \ln 2)} = e^{(\ln 4)} = 4$ (power law of $\ln$)，能得出精确值而非计算器近似值。

$$\begin{aligned} Q(20) &= 100,000 e^{(\ln 2)/10 \times 20} \\ &= 100,000 e^{(2 \ln 2)} \\ &= 100,000 e^{(\ln 4)} \\ &= 100,000 \times 4 \\ &= \$\blacksquare \end{aligned}$$

直觉核对： $k = (\ln 2)/10$ 意味着 doubling time (翻倍周期) = 10 年。 $\blacksquare$

Q1 — LINEAR / QUADRATIC / CVP

$ax^2 + bx + c \leq 0$ → factorise → roots → 开口向上取两根之间

$C(x) = cx + F$ → total cost = variable cost + fixed cost

$R(x) = sx$ → revenue = selling price × quantity

$P(x) = R(x) - C(x)$ → profit function

$R(x) = C(x)$ 或 $P(x) = 0$ → break-even point; 再代回 $R(x)$ 求 break-even revenue

Q2 — DEPRECIATION / EQUILIBRIUM

$\text{rate} = (P_0 - \text{residual})/\text{life}$ → annual depreciation (每年折旧额)

$V(t) = P_0 - (\text{rate}) \cdot t$ → linear depreciation, 注明 $0 \leq t \leq \text{life}$

$D(x) = S(x)$ → market equilibrium: demand = supply

$x = [-b \pm \sqrt{b^2 - 4ac}] / 2a$ → quadratic formula; 负根写 "(rejected, as x cannot be negative)"

Q3 — LOGS & EXPONENTIALS

$\log(MN) = \log M + \log N$ → product law

$\log M^n = n \log M$ → power law

$\log x = p \Leftrightarrow x = 10^p$ → log ↔ exponential form 互化

$e^{(kt)} = A \rightarrow kt = \ln A$ → taking ln on both sides

$e^{(\ln A)} = A$ → 化简 $e^{(2 \ln 2)} = e^{(\ln 4)} = 4$

§ 6 Common Pitfalls 常见扣分点 • Final Checklist

硬性扣分项 Hard mark-losers

- ① 无 working 直接给答案 → 0 分。
- ② Q2(c) 用 quadratic formula 而非题目指定的 factorisation。
- ③ Q1(a)(ii) 只写两个根 $x = -3, 7$, 没写区间 $-3 \leq x \leq 7$ 。
- ④ Q2(b) 负根未写拒绝理由 (as x cannot be negative)。
- ⑤ Q1(b)(ii) / Q2(b) 只求 quantity, 漏了 corresponding revenue / price。

格式与表述 Presentation

- ① 应用题开头定义变量 (Let x be ...)。
- ② 金额带 \$ 符号; 数量带单位 (units / toy cars / thousand units)。
- ③ Q2(a) 的 $V(t)$ 附定义域 $0 \leq t \leq 10$ 。
- ④ Q3 每一步注明所用 log law。
- ⑤ 非整数答案: 中间 4 d.p., 最终 2 d.p.。
- ⑥ Word 排版: Times New Roman 12, 每步等式独立一行。

提交前 Checklist

- ☐ 3 题全答, 每题以 "∴ ..." 结论句收尾
- ☐ Full name + student number 已写
- ☐ 每步方法与 lecture slides 一致 (Lecture 1-4 + Lecture on logs/exponentials)
- ☐ 所有数值代回原式验算过 (尤其 Q2(b)、Q2(c))
- ☐ 自留副本 • Canvas 按时提交

Academic integrity 提醒：本文件是学习参考，working 请用你自己的行文重写并确保每一步你都能独立复现——CA 卷面明确警告 plagiarism 会上报。逐字照搬他人（或 AI 生成）文本属高风险行为。

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